The Kolam Tradition

A tradition of figure-drawing in southern India expresses mathematical ideas and has attracted the attention of computer science

Marcia Ascher

Traditionally, the women of Tamil Nadu, in southeastern India, sweep their thresholds every morning, sprinkle them with a solution of cow dung and water and cover the area with elaborate, symmetrical figures using rice powder. They trickle the rice powder in a stream between their middle and index fingers, using their thumbs to guide the flow of the powder. According to tradition, the cow dung cleans and purifies the ground, and using the rice powder begins the day with an act of kindness by providing food for ants and other insects. Girls learn this kolam ritual from their female relatives, and kolam skills are viewed as a mark of grace and as a demonstration of dexterity, mental discipline and ability to concentrate. The kolam designs that appear daily on the thresholds of houses in Tamil Nadu attract attention from multiple perspectives. Beyond their significance in the Tamil culture, they are an unusual example of the expression of mathematical ideas in a cultural setting. Also, in recent years, kolam figures have attracted the attention of computer scientists interested in analyzing and describing images using picture languages.

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My own research has focused on exploring the diverse expressions and cultural settings of mathematical ideas. Complementing the historical development of modern mathematics, these studies are concerned with oral traditions and cultures that have not been viewed as part of the mathematical mainstream. As a guiding definition, I take mathematical ideas to be those involving number, logic, spatial configuration, and more significant, the organization of such ideas into systems and structures. Of particular interest are cases in which the people themselves view the ideas as important knowledge to be carefully learned and carefully passed on.

Using the criterion of mutually exclusive speech communities, at least 6,000 different cultures have existed during the past 700 years. Seeking mathematical ideas that were part of oral traditions most often involves reexamining, with a mathematical perspective, the work and materials of anthropologists, archaeologists, linguists and cultural historians. Some of the many cultural contexts of mathematical ideas include record keeping, calendar making, building, decoration, navigation, kinship organization, mapping, divination and religion. In essence, concepts of time and space and order placed upon the natural, supernatural and social worlds all may involve mathematical ideas. For example, the Incas developed a method of recording involving a numerical-logical system that used arrays of colored knotted cords; in the navigation tradition of the people of the Marshall Islands, two-dimensional constructions made of palm ribs tied with coconut fibers served as planar representations of their conceptualization of the interplay of wave swells and land masses; and divination in Madagascar relies on laying out arrays of seeds using a complex algebraic algorithm involving a two-valued logic. (Robert Ascher and I collaborated in 1981 on a book describing the Inca system; I have discussed the latter two examples, as well as other examples, in detail since then.)

What sets the kolam tradition apart from other cases I have studied is that it has contributed directly to an academic endeavor: The intricate figures have entered the realm of computer science. The figures have provided material for illustrating known approaches to the analysis and description of pictures and also have stimulated new approaches. This makes the kolam tradition a rare and especially interesting case.

The Kolam Figures

The kolam tradition of Tamil Nadu has persisted for hundreds of years and remains a common practice among women in cities as well as in rural areas, among those who are university-educated as well as those less schooled. In recent years, instead of rice, women have often substituted commercially available stone powder, chalk or ink for creating the designs, and women emigrating from Tamil Nadu often continue the practice in their new homes.

The daily placement of figures on the threshold is an important part of Tamil culture. The decorated threshold serves as a boundary between the inner and outer worlds, and the figures can both guard the household and welcome visitors to it. Women teach girls a range of figures and the procedures for drawing them as well as which figures are appropriate for everyday use and which

Figure 1. In Tamil Nadu, mothers teach their daughters how to draw kolam figures, which decorate thresholds. Learning to draw the figures is an important part of a girl's upbringing and has been for hundreds of years. In recent decades, the tradition has attracted the attention of computer scientists interested in analyzing and describing pictures using picture languages. The author, who has long studied the expression of mathematical ideas across cultures, describes how the kolam tradition has contributed to modern computer science. (Photograph courtesy of K. L. Kamat.)

are for special occasions or particular rituals and holidays. Learning the kolam tradition is an important part of a girl's upbringing. Although drawing kolam figures is an oral tradition passed on from mother to daughter, it is an oral tradition within a literate culture. Indeed, the Tamil have a written script and an extensive literature that goes back as far as the third or fourth century B.C. However, the Tamil literature only refers to the kolam tradition in passing and includes little detail. Still, the literature testifies to the longevity and ubiquity of the tradition. For example, one of the earliest known references, a 16th-century writing, tells about a peaceful and flourishing kingdom in which "the tiger and the cow drank from the same watering place, Brahmins chanted the Veda, women decorated the streets with *kolams*, rain fell on schedule, and the hungry were fed."

In parts of India outside Tamil Nadu, there are similar drawing traditions, among them ones called muggu, rangoli and alpana. Although the traditions are likely to be related historically, the figures differ, as do their meanings and the procedures for drawing them. In recent times, some of the traditions have expanded to include aspects of the others; as a result, sometimes the terms for them are used interchangeably. In this article, I will confine myself to considering kolam designs of the traditional form, namely figures made up only of white lines, or of white lines and dots, which are sometimes said to resemble netting, labyrinths or filigree.

Because the practice moves with people who emigrate from Tamil Nadu, the kolam tradition is found, for example, among tea estate workers in Sri Lanka, who arrived from Tamil Nadu in the late 19th century, and among those who emigrated to the United States. Deeply intertwined with the values, rituals and philosophy of the people of Tamil Nadu, the tradition serves as a marker of Tamil culture. This may be seen, for

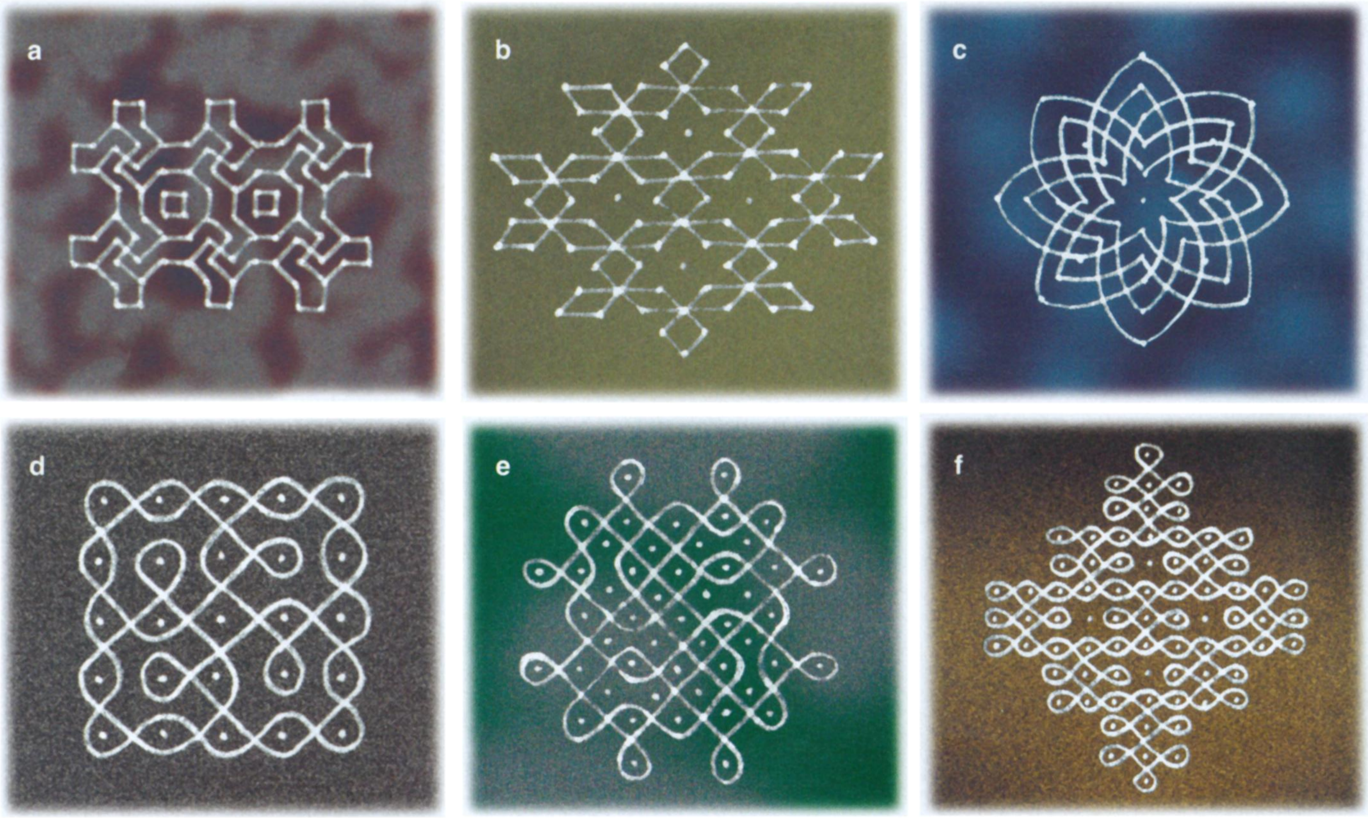


Figure 2. Kolam figures vary widely in style. All these examples began with a grid of dots; figures a, b and c connect the dots, whereas the curves in d, e and f go around the dots. Women draw figures c, e and f with closed continuous curves. Observers have noticed that d is not drawn continuously, although it could be drawn with a single closed curve. Note the different kinds of symmetry in the figures—symmetry around a horizontal centerline, around a vertical centerline and rotational symmetry around a center point of 45 degrees *(c)*, 90 degrees *(d)* and 180 degrees *(e)*.

example, by the choice of *Kolam* as the title of an international academic journal devoted to Tamil studies.

Many but not all kolam drawings begin with placing a grid of dots. The dots can, among other arrangements, make rectangular, triangular or hexagonal arrays or radiate from a central point. The figure is then drawn by connecting the dots or going around the dots, with the dots both guiding and constraining the drawing. For some of the kolam, which may or may not begin with a grid of dots, it is important to draw each figure with a single continuous line that ends where it began. These closed, continuous figures—and others made up of a few such closed lines—are associated with the never-ending cycle of birth, fertility and death, and with concepts of continuity, totality and eternity.

Observation shows that some figures that can be drawn as closed continuous curves are not actually drawn in one continuous sweep. For example, Figure 3 illustrates one such case; the kolam is drawn using systematic transformation of a basic unit. The figure repeats the basic unit four times, where each unit is rotated 90 degrees relative to the one before it. Finally, another continuous closed curve surrounds the four units. Collectively, the kolam figures demonstrate a marked concern for symmetry. Figures show symmetry around a horizontal line, a vertical line or various kinds of rotational symmetry.

Some kolam figures constitute families; that is, there are groups of figures that share common characteristics. In some cases, the larger figures in a family are made up of several joined copies of smaller figures; in other cases, the family members are derived from each other in more subtle ways. The conception and organization of families of kolam figures seem particularly expressive of mathematical ideas.

Picture Languages

The kolam figures grouped into families particularly attracted the interest of theoretical computer scientists concerned with the analysis and description of pictures through the use of picture languages, which use sets of basic units and specific, formal rules for combining the units. The study of picture languages is akin to formal language theory, which began some 45 years ago with Noam Chomsky's study of natural languages. In succeeding decades, computer scientists have used formal language theory in the analysis and specification of programming languages.

Gift Siromoney, of Madras Christian College in Tamil Nadu, initiated the use of kolam designs in the study of picture languages. A 1989 volume commemorating his life includes a bibliography that lists about 100 of Gift Siromoney's publications, a body of work that shows the many ways in which throughout his life he combined his academic interests with a deep concern for the environment, history and culture of Tamil Nadu. For Gift Siromoney, working with his wife Rani Siromoney, and for other computer scientists, in particular Kamala Krithivasan and K. G. Subramanian, the kolam designs became a rich source of figures that could be used as examples of some existing types of picture languages and also served as a stimulus for the creation of new types

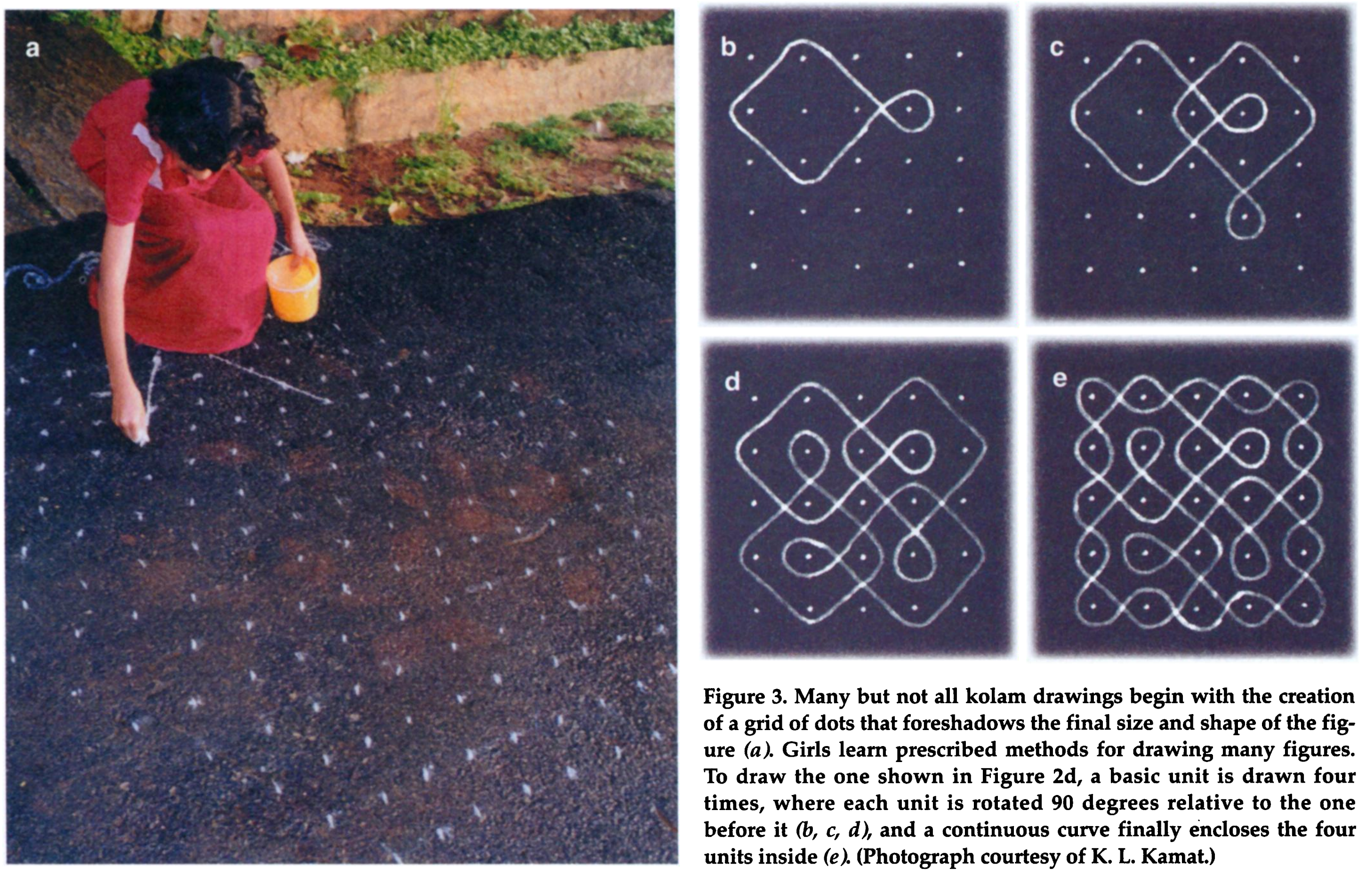


Figure 3. Many but not all kolam drawings begin with the creation of a grid of dots that foreshadows the final size and shape of the figure *(a)*. Girls learn prescribed methods for drawing many figures. To draw the one shown in Figure 2d, a basic unit is drawn four times, where each unit is rotated 90 degrees relative to the one before it *(b, c, d)*, and a continuous curve finally encloses the four units inside *(e)*. (Photograph courtesy of K. L. Kamat.)

of languages. Other computer scientists in addition to the Madras group have used picture languages to describe kolam families.

Here I will describe picture languages used to produce just a few kolam families. As a prelude, let's look at an example of a rudimentary formal language that produces strings of symbols and then see how such strings can be translated into pictures. Let us say that the symbols in our language are restricted to A, B, C and the starting string is ABAA. Our rules for creating a new string of symbols from a previous string will be:

$$B \rightarrow AC,\ A \rightarrow B,\ C \rightarrow CC.$$

We refer to the strings that result from applying these rewriting rules to the previous string as *outcomes*. Accordingly, if the starting string is ABAA, the first outcome will be BACBB, the second outcome will be ACBCCACAC, and the third outcome will be BCCAC CCCCBCCBCC. The rewriting rules can continue to generate outcomes indefinitely. At each stage, the three rules are applied simultaneously—that is, in parallel—rather than sequentially, so, for example, the A and C introduced at one stage by applying the rule B→AC remain unmodified until a later stage. This is characteristic of a Lindenmayer

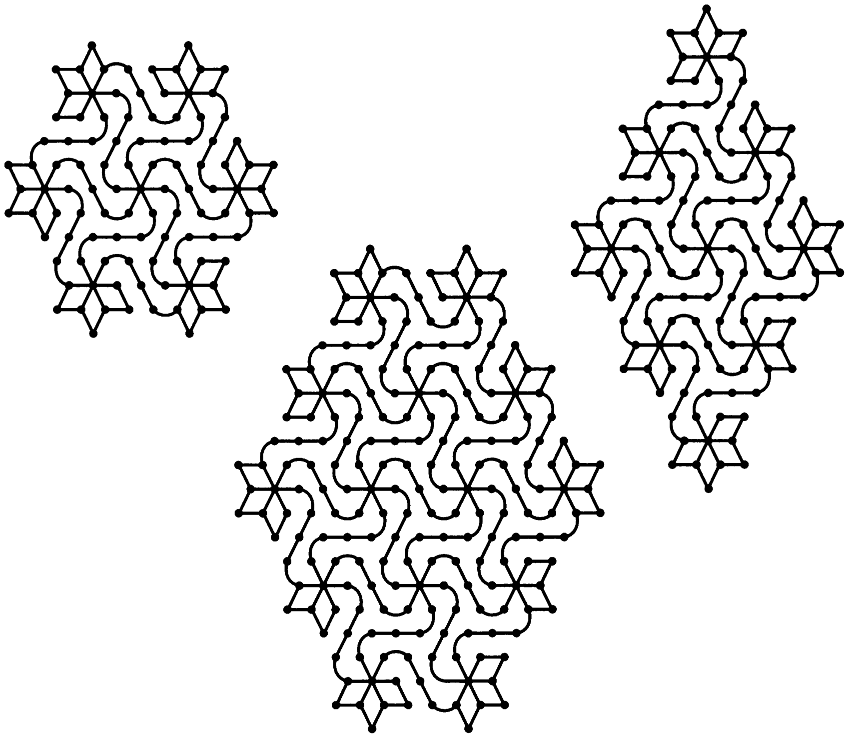


Figure 4. Some kolam figures constitute families with family members that share common characteristics or derive from each other. These three kolam are members of the Parijatha Creeper family. The creation of families is particularly expressive of mathematical ideas. Computer scientists have sought to develop picture languages capable of describing kolam families.

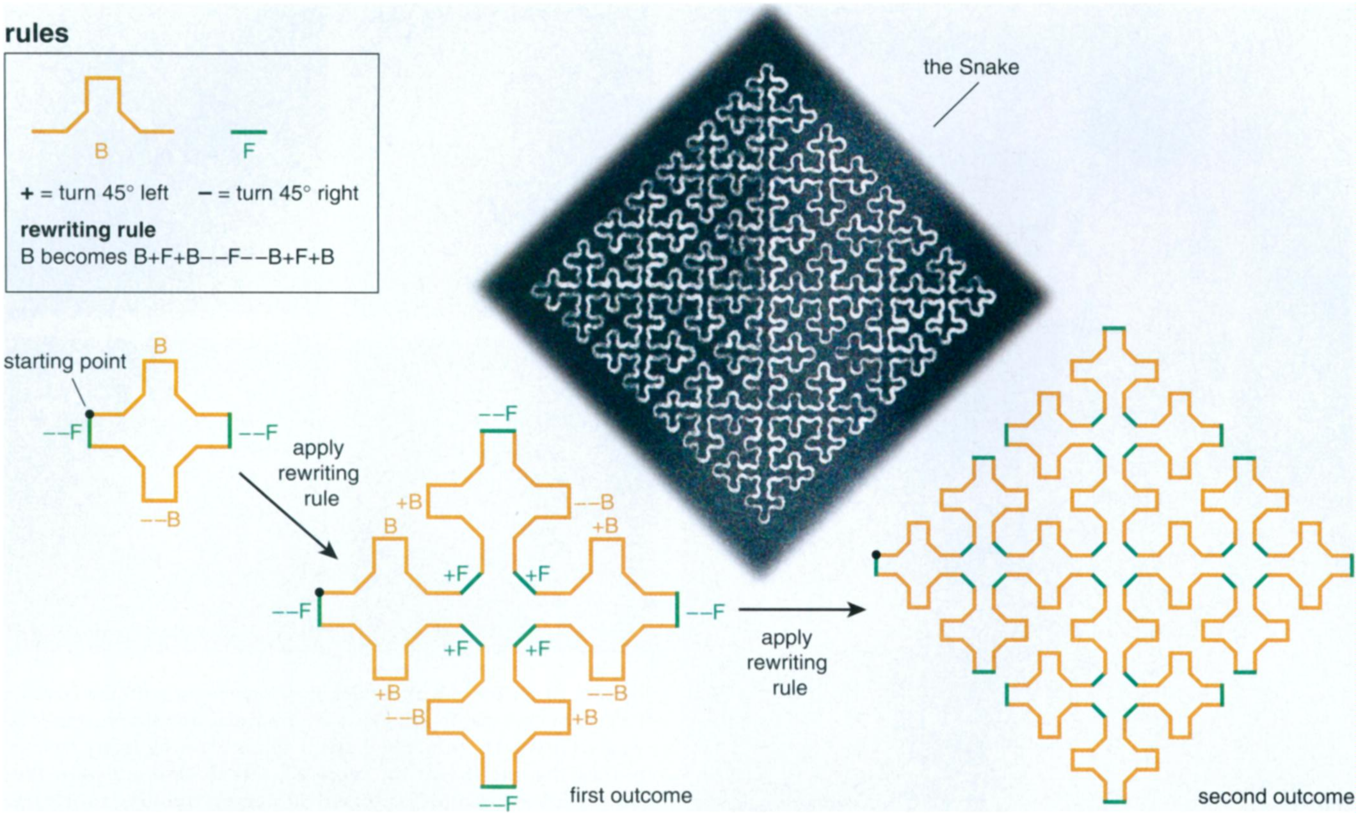


Figure 5. A picture language can create angular versions of a kolam type called the Snake. The smallest figure shows a starting string of B– –F– –B– –F, where F means move ahead by one step, – means move clockwise by 45 degrees, + means move counterclockwise by 45 degrees and B means F+F+F- -F- -F+F+F. Applying the rewriting rule B→B+F+B– –F– –B+F+B produces more complex versions of the Snake kolam. The first outcome is (B+F+B– –F– –B+F+B)– –F– –(B+F+B– –F– –B+F+B)– –F. Each application of the rewriting rule replaces each arm in the four–armed cross with a new four-armed cross, causing the figures to grow exponentially from 4 to 16 to 64 arms, and so on.

language, or *L-language*—named for Aristid Lindenmayer, a biologist interested in modeling plant growth. In our example, the language is a context-free, deterministic L-language. It is context-free because the fate of each symbol is dealt with individually, without reference to the neighboring symbols, and deterministic because there is only one possible rewriting rule for each of the symbols.

Now the question is how to create a picture from a symbol string. Przemsylaw Prusinkiewicz, who is particularly noted for computer-graphics applications of L-systems, developed techniques using symbols interpreted as "turtle commands." Turtle graphics were the idea of Seymour Papert in the 1960s; he conceived the language primarily to foster the imagination of children through the use of computers. The idea is that a turtle can draw an image by dragging its dirty tail. Or it can lift its tail to create a discontinuous drawing. The turtle does not have an overall perspective, but it can create complex drawings by understanding and carrying out a limited set of commands that are conveyed to it by a set of symbols. Those commands include:

F: Move forward by a step while drawing a line.
f: Move forward by a step without drawing a line.
+: Turn left (counterclockwise) by an angle of *d* degrees.
–: Turn right (clockwise) by an angle of *d* degrees.

Each drawing must begin by specifying the turtle's direction and the angle *d*, which remains fixed throughout the drawing. Each move begins in the place and direction in which the last one ended. As an example, a context-free, deterministic L-language with turtle interpretation can reproduce the Snake family of kolam figures (*Figure 5*). Drawn continuously and ending where it began, a Snake kolam differs from many other kolam in that no dots precede its drawing. The turtle interpretation of it uses straight lines and produces an angular version of the design, though incorporating a mathematical data-fitting technique can, instead, generate a smooth curve. In any case, a language that describes the angular Snake begins by defining *d* as 45 degrees, and takes B– –F– –B– –F as a starting string. The turtle always interprets B to mean F+F+F– –F– –F+F+F, and the rewriting rule for producing more complex Snake figures is B→B+F+B– –F– –B+F+B. Figure 5 shows pictorial representations of B, of the starting string and of the first and second outcomes. Successive applications of the rewriting rule cause the design to grow exponentially. The most intricate Snake kolam shown is a smoothed version of what would be the third outcome. Were we to reduce the size of the turtle's steps for each outcome so that successive drawings would fit into a square of the same size, and were we to continue indefinitely, we would have as a surprise result a variant of the space-filling, self-avoiding fractal known as the Sierpinski curve, named for the mathematician who described it in 1912.

A context-free, deterministic L-language has also been used to describe another kolam family—the Anklets of Krishna. As in the Snake family, the members of the Anklets of Krishna family are derived recursively from each other and grow exponentially. The Madras group of computer scientists avoided producing angular versions of the kolam figures by using what they termed *kolam moves* to draw smooth curves and loops instead of using linear turtle moves. The Madras group defined seven kolam moves based on Tamil women's descriptions of their actions. Drawing the Anklets of Krishna requires only three of the kolam moves:

F: Move forward while drawing a line.
R_1: Move forward while making a half turn to the right.
R_2: Move forward while making a full loop to the right.

A language that produces the Anklets of Krishna begins with the starting string $R_1FR_2FR_2FR_2FR_1$ and has the following rewriting rules: $R_1 \rightarrow R_1FR_2FR_1$ and $R_2 \rightarrow R_1FR_2FR_2FR_2FR_1$ (*Figure 6*). Like the Snake kolam, where each application of the rewriting rule replaces each arm in the four-armed cross with a new four-armed cross, increasingly intricate Anklets of Krishna figures are produced by replacing each leaflet with a set of four leaflets. Several kolam families grow in a similar way.

Array Languages

The picture languages I have described can express some families of figures, but they ignore the dot arrays with which most kolam figures begin. The dot arrays that the women of Tamil Nadu lay out before drawing kolam foreshadow the final size and shape of each figure. Or, put another way, with the dots, the women create the two-dimensional skeletal structures of figures before executing the drawings. This notable aspect of the kolam tradition motivated the Madras group to work on *array grammars*, which deal with two-dimensional arrays of symbols instead of strings of symbols. Beginning with arrays of symbols, the rewriting rules of array languages replace subarrays with other subarrays. But one recurrent problem that they had to resolve is that the newly introduced subarrays can differ in shape or size from the subarrays they replace, creating shape distortions in the new array. In the array grammars referred to as Siromoney matrix grammars, simple transformations of figure units can be expressed. This could apply, for example, to the generation of the family of the kolam in Figure 2a. That figure has the same unit reproduced two times horizontally, a single time vertically. The basic unit might, in another kolam, be reflected while being repeated. Another type of grammar, the "kolam array grammars," deals with the generation of figures where the length and width of the figures have a fixed relationship. Important to the contribution of the Madras group is their generalization of some basic string operations in order to apply them to rectangular arrays and also to hexagonal arrays. In a 1986 article, Rani Siromoney provides a comprehensive discussion and bibliography of the Madras group's extensive work on array grammars; in it, she describes the various facets that the group pursued and how their work relates to those of others. Most of their work deals with the formal properties of the types of languages they proposed, while some of their writings apply the languages to kolam families.

The Madras group used two distinct methods for pictorial interpretation of the symbolic arrays that the array languages generate. One method interprets the symbols in rectangular arrays as contiguous figure units. The specific sets of figure units vary from language to language, since they depend on the kolam family that the array language describes. To create various members of a kolam family, the rules for generating successive arrays have to capture the inherent organization of the figure units in a given family. The second method for producing pictures from the symbolic arrays moves closer to the procedures the women of Tamil Nadu use to draw kolam. In it, the symbols in the arrays can be thought of as dots, but dots that carry instructions guiding the drawing of the figures. The types of dots in an array and their instructions vary from language to language-

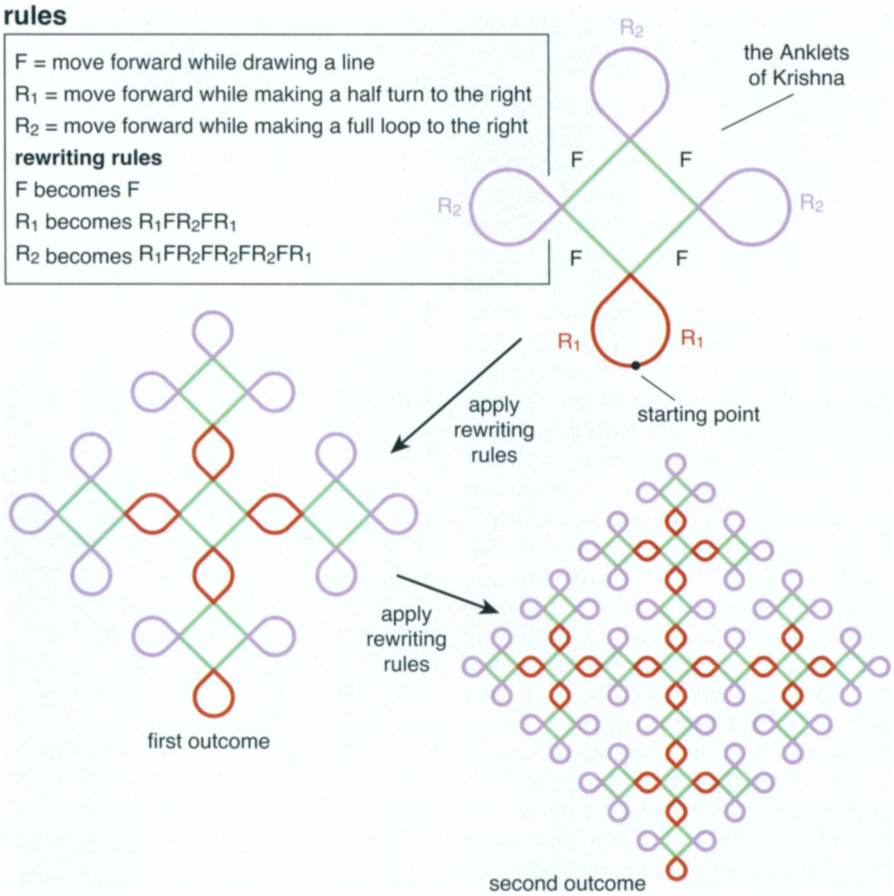
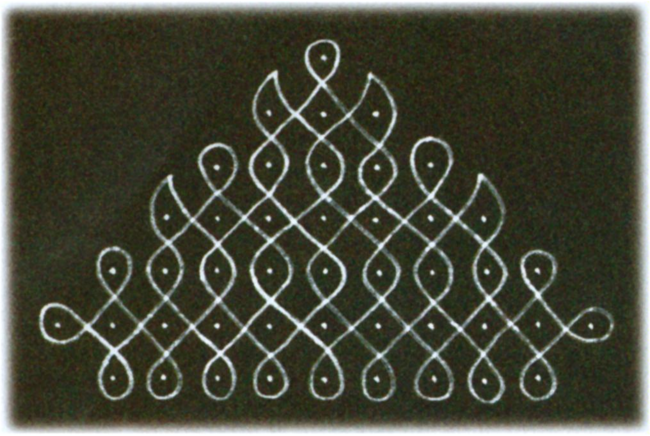


Figure 6. A picture language creates kolam figures in the Anklets of Krishna family using the starting string $R_1FR_2FR_2FR_2FR_1$ and the boxed rules that dictate what each symbol means and how to rewrite the symbols in the string to create increasingly intricate outcomes. Each application of the rewriting rules replaces each of the leaflets with a set of four leaflets.



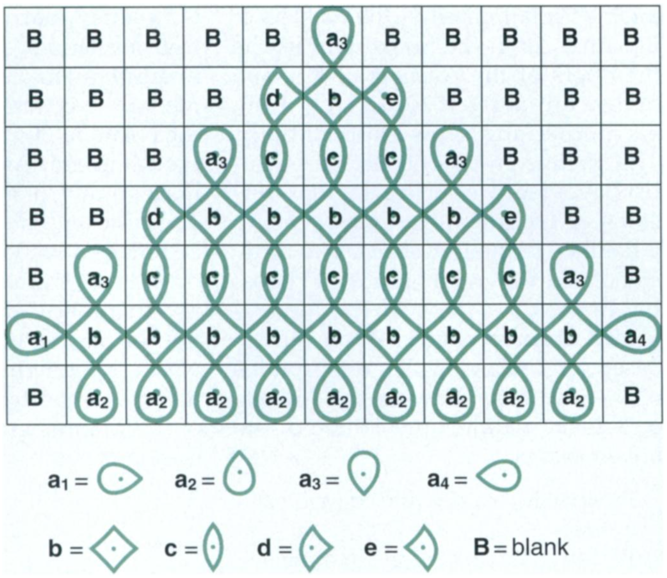


Figure 7. Different array languages can create the Mountain Top family of kolam figures, which grows polynomially rather than exponentially. In one language, the symbols in the resulting rectangular arrays are interpreted as contiguous figure units. At right, the figure units are superimposed on a symbolic array.

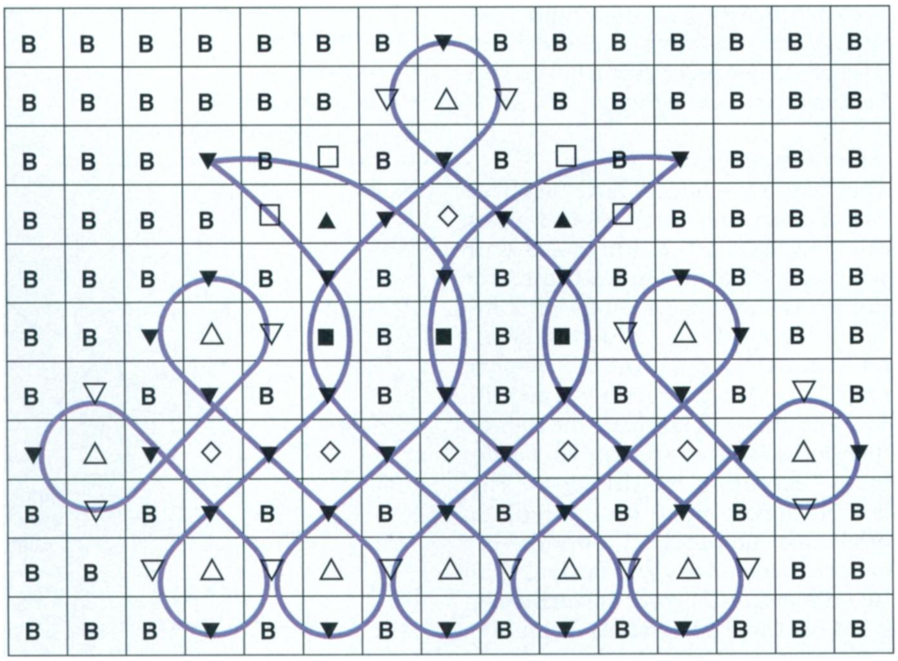
guage; again, they are specific to the kolam family that the arrays describe. To create kolam family members, the rules for generating successive arrays must capture the patterned organization of the instruction-carrying dots. Figures 7 *(right)* and 8 show how the two different methods can generate Mountain Top kolam figures.

The Algorithmic Nature of Kolam

As computer scientists have tried to capture kolam figures with picture languages, they have highlighted the richness of the kolam structures and their algorithmic nature—in other words, the orderly, step-by-step way they are built. The languages do not necessarily replicate how the women of Tamil Nadu conceive of and draw the kolam figures. Nevertheless, they underscore the fact that the kolam, and particularly the families of kolam, are more than just a collection of individual pictures; systematic procedures and techniques unify them.

The kolam tradition has also provided an unusual opportunity for the computer scientists. There may be no better way to examine an academic construct than to apply it to examples from a tradition and culture outside the one in which the construct arose. But in addition, the computer scientists sought to learn from the figure makers and integrated what they learned into the theory and practice of their own field. This recent phase in the history of the kolam tradition showcases how mathematical ideas can move beyond their traditional boundaries, interact with an academic endeavor, and in fact contribute to it.

There is no doubt that mathematical ideas are embedded in the kolam tradition, with its emphasis on rich symmetry, patterned repetition, closed continuous curves and families of curves. Individual women conceive of and execute the traditional kolam figures and introduce variations among them, but they are, nonetheless, part of a corpus of ideas passed on from generation to generation. And the figures are an important part of the cultural environment in Tamil Nadu—seen, known and appreciated by everyone in the culture. The kolam tradition is certainly part of a global and ongoing history of mathematical ideas, yet it remains, above all, a central part of the culture and daily landscape of Tamil Nadu.



rules

1. join two ▽ dots by an arc through a ▼ dot around a △ dot
2. join two nearest ▼ dots by straight lines (to form diamonds around ◇ dots)
3. join ▼ dots around a ■ dot by two arcs
4. join two ▼ dots through a □ dot by a thread

Figure 8. Another array language that generates members of the Mountain Top family of kolam figures interprets the symbols in the arrays as dots carrying specific instructions.

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Bibliography

Ascher, M. 1994. *Ethnomathematics: A Multicultural View of Mathematical Ideas*. New York: Chapman & Hall/CRC.

Ascher, M. In press. *Mathematics Elsewhere*. Princeton, N.J.: Princeton University Press.

Ascher, M., and R. Ascher. 1997. *Mathematics of the Incas: Code of the Quipu*. New York: Dover Publications, Inc. (Reprint of 1981 *Code of the Quipu: A Study in Media,* Mathematics, *and Culture*. Ann Arbor: University of Michigan Press.)

Durai, H. G. 1929. Preliminary note on geometrical diagrams (kolam) from the Madras Presidency. *Man* 29:77–78

Narasimhan, R. 1989. List of selected publications of Dr. Gift Siromoney. In *A Perspective in Theoretical Computer Science: Commemorative Volume for Gift Siromoney*, ed. R. Narasimhan. Series in Computer Science, vol. 16. London: World Scientific, pp. xi–xviii.

Narasimhan, R. 1992. The oral-literate dimension in Indian culture. In *Indological Essays: Commemorative Volume II for Gift Siromoney*, ed. M. Lockwood, Department of Statistics, Madras Christian College, pp. 67–79.

Prusinkiewicz, P., K. Krithivasan and M. G. Vijayanarayana. 1989. Application of L-systems to algorithmic generation of South India folk art patterns and Karnatic music. In *A Perspective in Computer Science: Commemorative Volume for Gift Siromoney*, ed. R. Narasimhan. Computer Science Series, vol. 16. London: World Scientific, pp. 229–247.

Prusinkiewicz, P., and A. Lindenmayer. 1990. *The Algorithmic Beauty of Plants*. New York: Springer-Verlag.

Prusinkiewicz, P., A. Lindenmayer and F. D. Fraccia. 1990. Synthesis of space-filling curves on the square grid. In *Fractals in the Fundamental and Applied Sciences*, eds. H.-O. Peitgen, J. M. Henriques and L. F. Penedo. Amsterdam: North-Holland Press, pp. 341–366.

Rosenfeld, A., and R. Siromoney. 1993. Picture languages—a survey. *Language of Design* 1:229–245.

Siromoney, G. 1978. South Indian kolam patterns. *Kalakshetra Quarterly* 1:9–15.

Siromoney, G., and R. Siromoney. 1987. Rosenfeld's cycle grammars and kolam. In *Graph-Grammars and their Application to Computer Science*, Third International Workshop 1986, ed. H. Ehrig, M. Nagl, A. Rosenfeld and G. Rozenberg. Lecture Notes in Computer Science, no. 291. Berlin: Springer-Verlag, pp. 565–579.

Siromoney, G., R. Siromoney and K. Krithivasan. 1974. Array grammars and kolam. *Computer Graphics and Image Processing* 3:63–82.

Siromoney, R. 1986. Array languages and Lindenmayer systems. In *The Book of L*, eds. G. Rozenberg and A. Saomaa. Heidelberg: Springer-Verlag, pp. 413–426.

Steinmann, R. M. 1989. Kolam: Form, technique, and application of a changing ritual folk art of Tamil Nadu. In *Shastric Traditions in Indian Arts*, ed. A. L. Dallapiccola. Stuttgart: Steiner, vol. 1, pp. 475–491, vol. 2, pp. 131–135, and plates C–CIV.

Links to Internet resources for further exploration of "The Kolam Tradition" are available on the *American Scientist* Web site:

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